Incentives Models of Cost Progress

With An Application to Electronics Systems

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Where we've been

- Ad hoc models of cost progress, e. g. $C_j = T_1 j^b \left(\frac{R}{R_0} \right)^c$
- One curve shape parameter, b or, equivalently, slope $S = 2^b$
- For initial estimates, choose S by commodity, e. g. for a/c S ~ 80%, for electronics S ~ 90%

Can we do better?

- Rational model of cost progress
- Relate features of cost progress model to features of product, plant, and, perhaps, industry

Where we're going

- The idea that cost progress comes mostly from investments that either make items cheaper to produce, or make plants more efficient, leads to a three-parameter cost progress model
- The parameters relate naturally to characteristics of the product, and of the production operation
- Analysts may find this approach useful for estimating cost progress, either qualitatively or quantitatively

First steps

Model cost progress as the payoff of investments in producibility and production technology.

Determine investment patterns as responses to economic incentives.

To model cost progress, ask why it happens:

- Workers learn to do their jobs better
- Products are re-designed to make them cheaper to produce
- Production technology is improved
- Cheaper sources of inputs are identified

All but the first of these are results of investment. So, model unit cost C as function of investment I: C = f(I)

A not-quite-arbitrary choice of the variation of unit cost with investment

Let unit cost C vary with investment I as $C = f(I) = C^* + \Delta e^{-\alpha I}$

Derivation: $\partial C = -\alpha \partial I (C - C^*)$

$$\frac{d(C - C^*)}{C - C^*} = -\alpha \, dI$$

$$C(0) = C_0$$

$$C = C^* + (C_0 - C^*) e^{-\alpha I}$$

This f(I) builds in diminishing returns, and a minimum unit cost

Investment incentives model

Management's general problem: Choose period-by-period investment stream $\delta_0, \delta_1, \ldots, \delta_{M-1}$ to maximize profit. Leads to maximizing

$$P = \Big\{\big[N_0\big(p_0 - f(0)\big) - \delta_0\big] + \big[N_1\big(p_1 - f(\delta_0)\big) - \delta_1\big] + \big[N_2\big(p_2 - f(\delta_0 + \delta_1)\big) - \delta_2\big]$$

$$+ \ldots$$

$$+ \big[N_{M-1}\big(p_{M-1} - f(\delta_0 + \delta_1 + \ldots + \delta_{M-2})\big) - \delta_{M-1}\big] + \big[N_M\big(p_M - f(\delta_0 + \delta_1 + \ldots + \delta_{M-1})\big)$$

(or, perhaps, a NPV version of this).

A simple possibility

- Management objective is simply to minimize cost
- At each lot i, choose investment δ_i to minimize cost-to-go
- Each period's investment is limited: $\delta_i \le \delta_{max}$ (inventing and implementing product and plant improvements takes time; capital rationing)

Minimization problem

$$\min_{\delta_i}\left\{\sum_{j=i+1}^{M} N_j f(I_{i-1}+\delta_i)+\delta_i\right\} \ni \delta_i \le \delta_{max}$$

which is the same as

$$\min_{\delta_i}\left\{QR_i f(I_{i-1}+\delta_i)+\delta_i\right\} \ni \delta_i \le \delta_{max}$$

N_j = quantity for lot j; M is total number of lots; $QR_i \equiv \sum_{j=i+1}^{M} N_j$

Solution:

$$\delta_i = \begin{cases} \min\left(\dfrac{1}{\alpha}\ln(QR_i\alpha\Delta) - I_{i-1}, \delta_{max}\right), \text{if } \dfrac{1}{\alpha}\ln(QR_i\alpha\Delta) - I_{i-1} > 0 \\ 0, \text{else} \end{cases}$$

where QR_i denotes $(Q - Q_i)$.

Resulting simple unit cost profile

$$
\begin{aligned}
C_0 &= C^* + \Delta \\
C_1 &= C^* + \Delta e^{-\alpha\delta_{max}} \\
C_2 &= C^* + \Delta e^{-2\alpha\delta_{max}} \\
\cdots &\qquad \cdots \\
C_{i^*-1} &= C^* + \Delta e^{-(i^*-1)\alpha\delta_{max}} \\
C_{i^*} &= C^* + 1/(\alpha QR^*) \\
C_{i^*+1} &= C^* + 1/(\alpha QR^*) \\
\cdots &\qquad \cdots
\end{aligned}
$$

where i* is the first i such that

$$\frac{1}{\alpha}\ln(QR_i\,\alpha\Delta) - (i-1)\delta_{max} \leq \delta_{max}$$

or

$$i \geq \frac{1}{\alpha\delta_{max}}\ln(QR_i\,\alpha\Delta)$$

Resulting simple unit cost profile

$$C_0 = C^* + \Delta$$

$$C_1 = C^* + \Delta e^{-\alpha\delta_{max}}$$

$$C_2 = C^* + \Delta e^{-2\alpha\delta_{max}}$$

$$\ldots \qquad \ldots$$

$$C_{i^*-1} = C^* + \Delta e^{-(i^*-1)\alpha\delta_{max}}$$

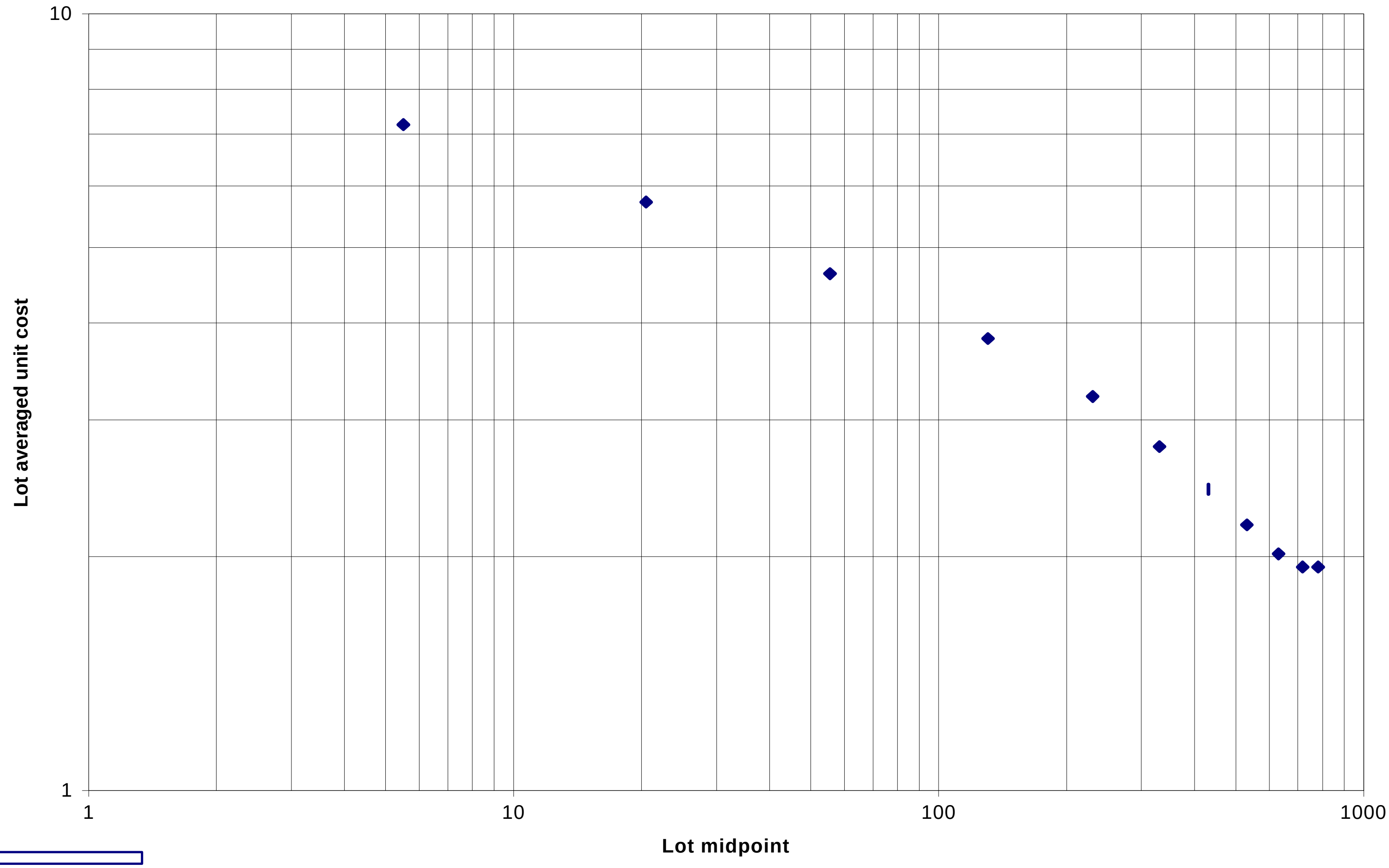
$$C_{i^*} = C^* + 1/(\alpha Q R^*)$$

$$C_{i^*+1} = C^* + 1/(\alpha Q R^*)$$

$$\ldots \qquad \ldots$$

Parameters and buy pattern determine profile; investment does NOT appear!

Example



A significant difference from power-law cost-progress curves:

Unit cost never falls below

$$C_{\min} \equiv C^* + 1/\alpha Q R^*$$

and the ratio of unit cost to initial unit cost is never smaller than

$$R_0 \equiv \frac{1 + 1/(\alpha Q R^* C^*)}{1 + \Delta}$$

Curve has three shape parameters

$H \equiv \frac{\Delta}{C^*}$ **("Headroom"; measures excess of initial cost over best cost)**

$S \equiv \alpha \overline{N} C^*$ **("Sensitivity"; ratio of "good" lot cost to e-folding investment)**

$L \equiv \alpha \delta_{max}$ **("Limit"; ratio of maximum investment to e-folding investment)**

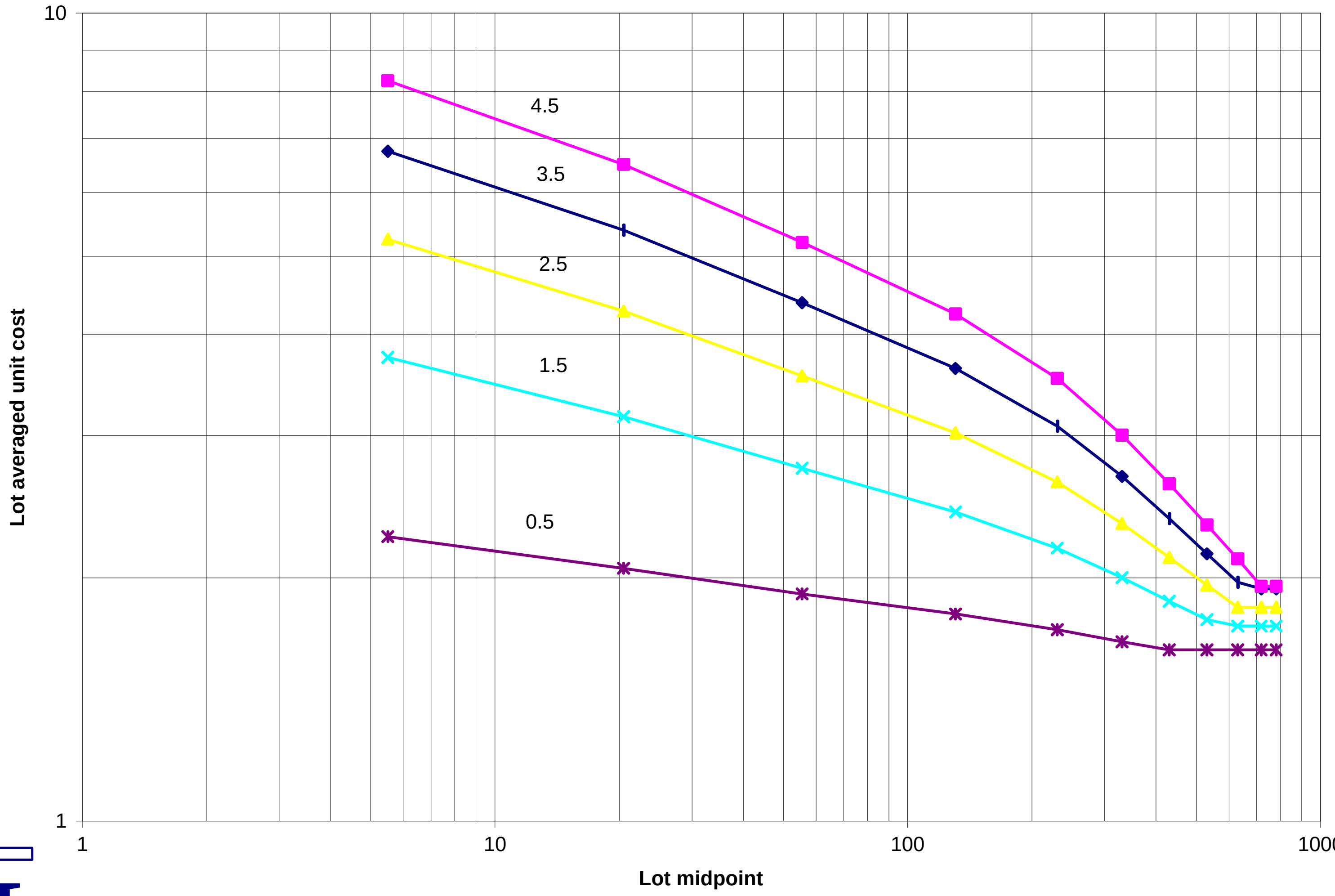
These, together with buy profile and the value of C*, determine the cost progress curve

Three-parameter cost progress model

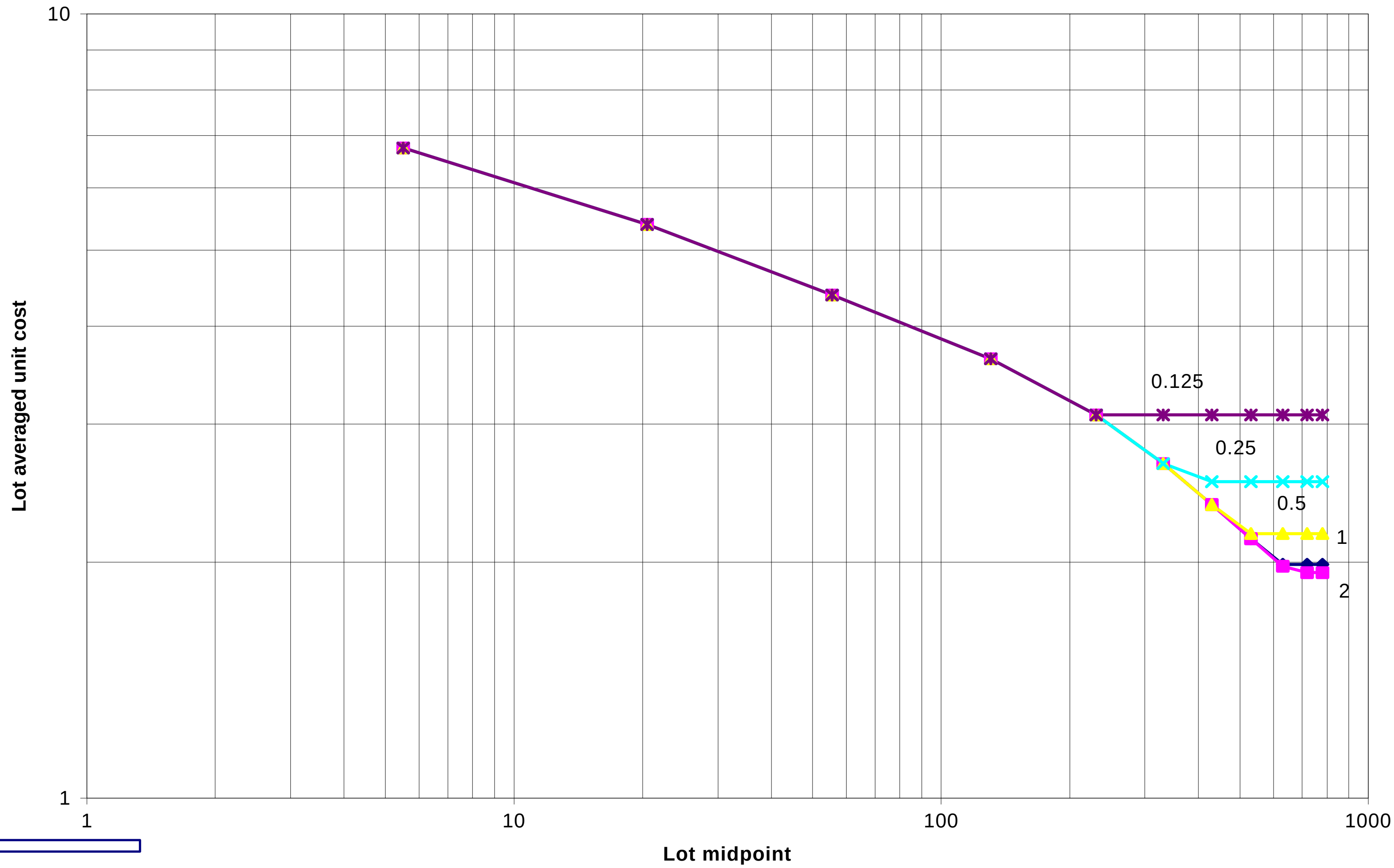
$$C_i = \begin{cases} C^*\left[1 + He^{-iL}\right], & 1 \le i < i^* \\ C^*\left[1 + \dfrac{\overline{N}}{QR_{i^*}S}\right], & i \ge i^* \end{cases}$$

$$i^* \equiv \max\left\{ i \mid \frac{1}{L}\ln\left(\frac{QR_i}{\overline{N}}SH\right) \ge i \right\}$$

Varying headroom H



Varying sensitivity S



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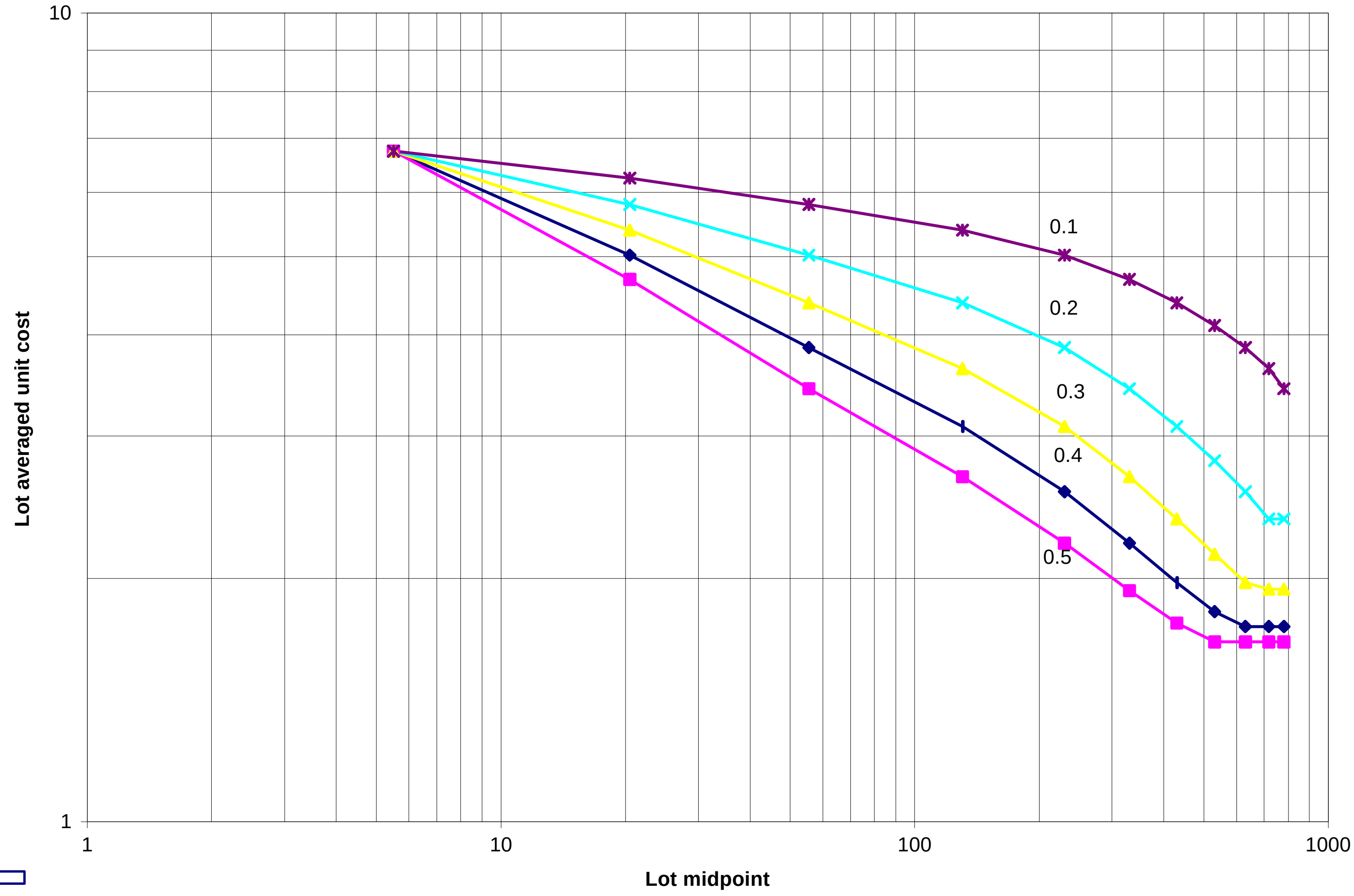
Varying limit L



Qualitative relations of parameters to product, production characteristics

$$\boxed{H \equiv \frac{\Delta}{C^*}}$$

Leads to larger H

- Hurried EMD; great time pressure for item
- Firm has little experience producing similar items

Leads to smaller H

- Substantially automated plant

H is large when production begins at unit cost well above best unit cost

Qualitative relations of parameters to product, production characteristics

$$\boxed{S \equiv \alpha \overline{N} C^*}$$

Leads to larger S

- Flexible, relatively inexpensive tooling
- Many steps in production

Leads to smaller S

- Extensive, expensive specialized tooling
- Substantially automated facility

S is large when lot cost is large compared to e-folding investment

Qualitative relations of parameters to product, production characteristics

$$\boxed{L \equiv \alpha\delta_{max}}$$

Tends to larger L

- Product dominant in firm
- Competition or threat thereof
- Great confidence in total quantity

Tends to smaller L

- Sole-source procurement
- Uncertain future

Quantitative relations of parameters to product, production characteristics

- Three binary variables:
  - f_1: 1 => "complex" product
  - f_2: 1 => "automated" manufacturing
  - f_3: 1 => "competition" or threat thereof

Values

System	f1 (complex?)	f2 (automated?)	f3 (competition?)
AN/MPQ-53	1	0	0
AN/APG-71	1	0	0
FAA ASR-9	1	1	0
SQQ-89	1	0	1
AEGIS	1	0	0
SINCGARS-ITT	0	0	1
SINCGARS-GD	0	1	1
PLGR	0	1	0

Relating curve parameters to product and plant

Three translog functions:

$$H = H_0 \beta_1^{f_1} \beta_2^{f_2} \beta_3^{f_3}$$

and similar translog functions for S and L.

With C* and rate exponent c for each system, we have 12 + 16 = 28 adjustable parameters. Our data are 45 values of lot-averaged unit costs.

All-up cost progress model

$$C_i = \begin{cases} C * \left[1 + H(\vec{f})e^{-iL(\vec{f})}\right]\left(\dfrac{N_i}{N_{ref}}\right)^c, & 1 \le i < i^* \\ C * \left[1 + \dfrac{\overline{N}}{QR_{i^*}S(\vec{f})}\right]\left(\dfrac{N_i}{N_{ref}}\right)^c, & i \ge i^* \end{cases}$$

$$i^* \equiv \max\left\{ i \mid \frac{1}{L}\ln\left(\frac{QR_i}{\overline{N}}S(\vec{f})H(\vec{f})\right) \ge i \right\}$$

$$\vec{f} \equiv (f_1, f_2, f_3)$$

To get lot-cost sequence, besides usual T_1, lot sizes, and c, answer three questions to evaluate f_1, f_2, and f_3)

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Resulting parameters

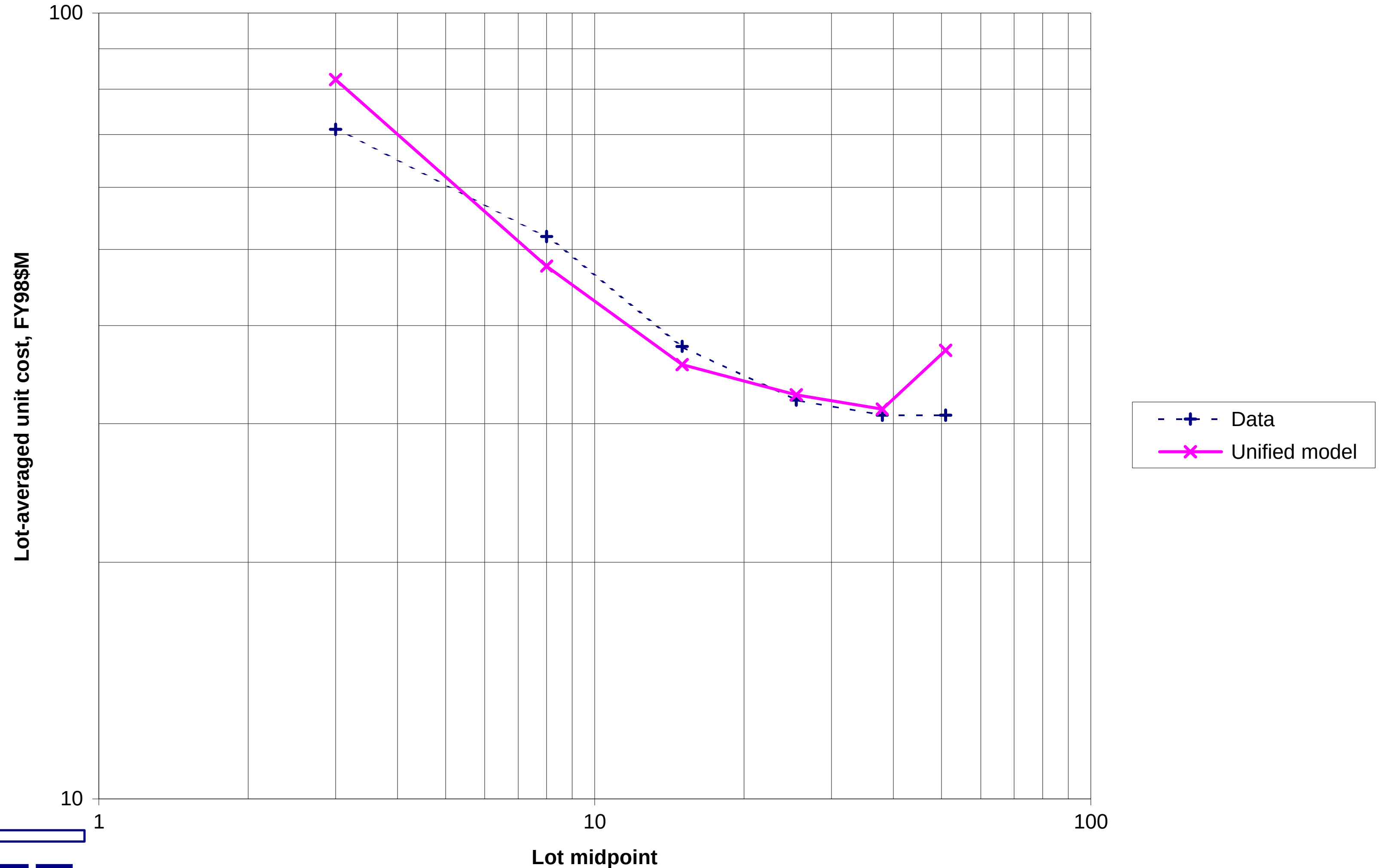
System	f1 Complex?	f2 Automation?	f3 Competition?	H	S	L
AN/MPQ-53	1	0	0	1.62	1390	10.5
AN/APG-71	1	0	0	1.62	1390	10.5
FAA ASR-9	1	1	0	0.18	36.3	0.34
SQQ-89	1	0	1	2.19	699	0.35
AEGIS	1	0	0	1.62	1390	10.5
SINCGARS-ITT	0	0	1	2.00	62.8	0.29
SINCGARS-GD	0	1	1	0.23	1.64	0.009
PLGR	0	1	0	0.168	3.26	0.26

For U. S. Government People

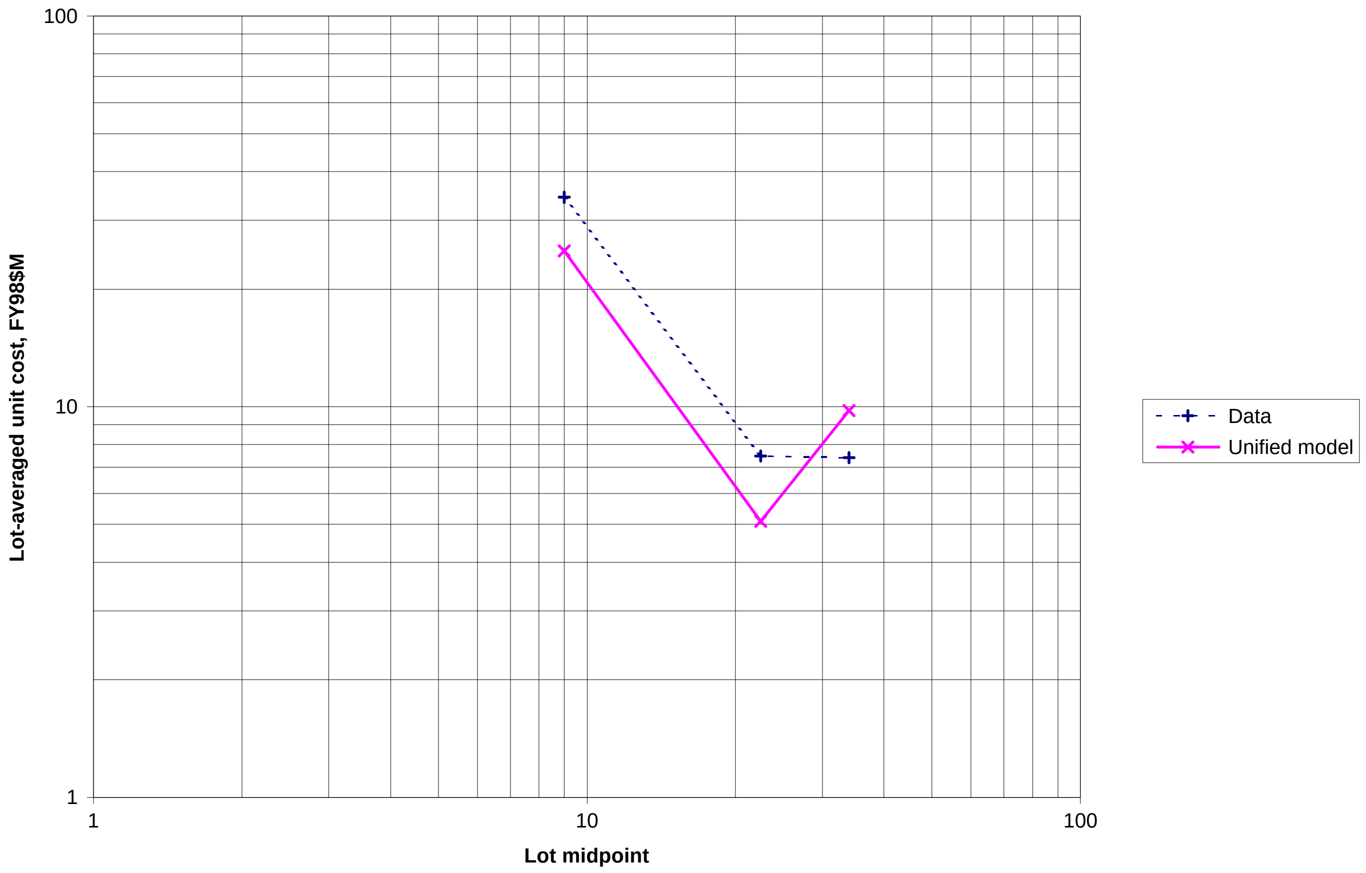
Product names were removed from the following charts, because some of the data are proprietary. U. S. Government personnel may receive the unedited charts, by requesting them by e-mail to

dlee@lmi.org

Results


100
10
1
10
100
Lot-averaged unit cost, FY98$M
Lot midpoint
Data
Unified model

Results



100
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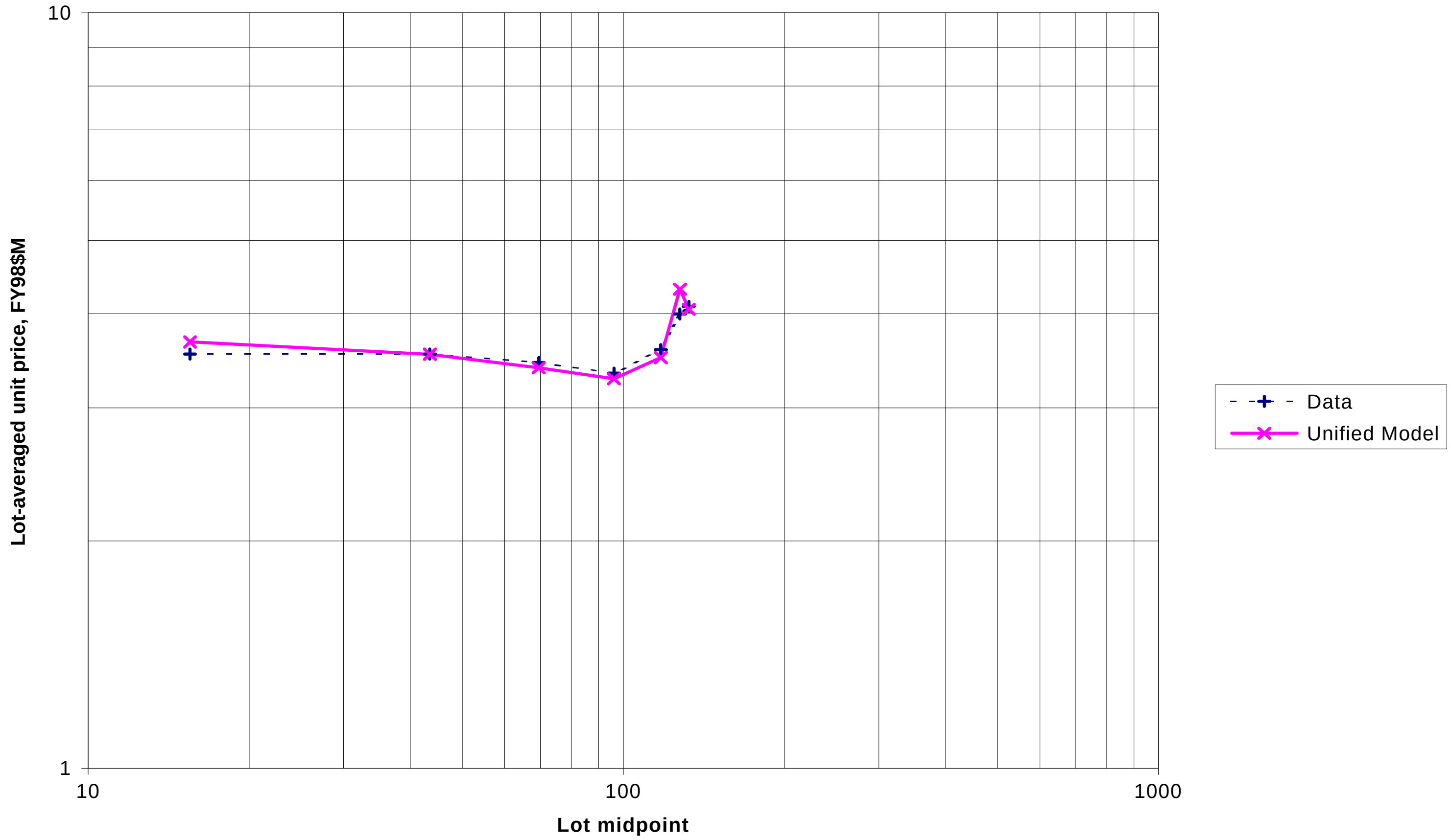
Lot-averaged unit cost, FY98$M
Lot midpoint
Data
Unified model

Results



10
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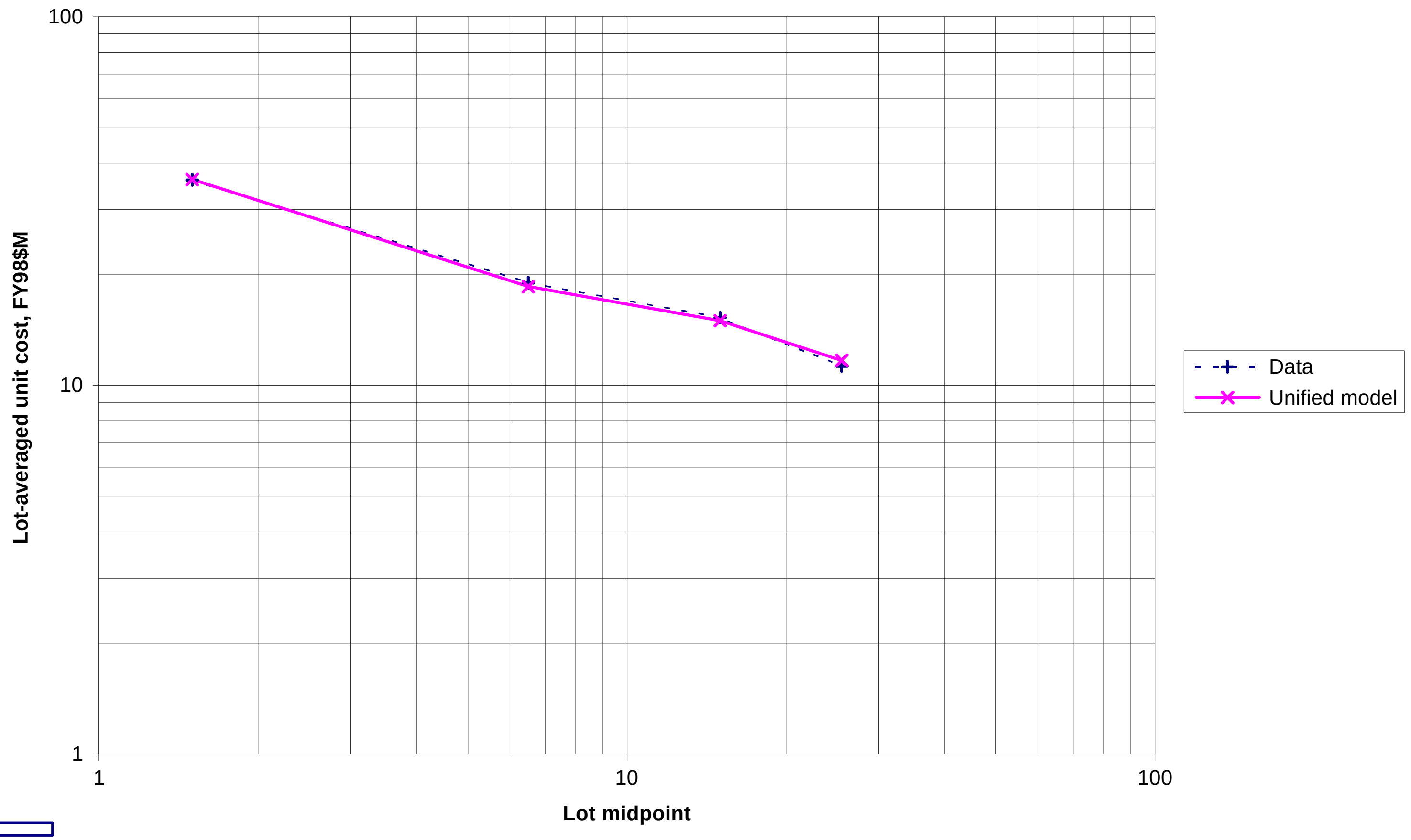
Lot-averaged unit price, FY98$M
Lot midpoint
Data
Unified Model

Results



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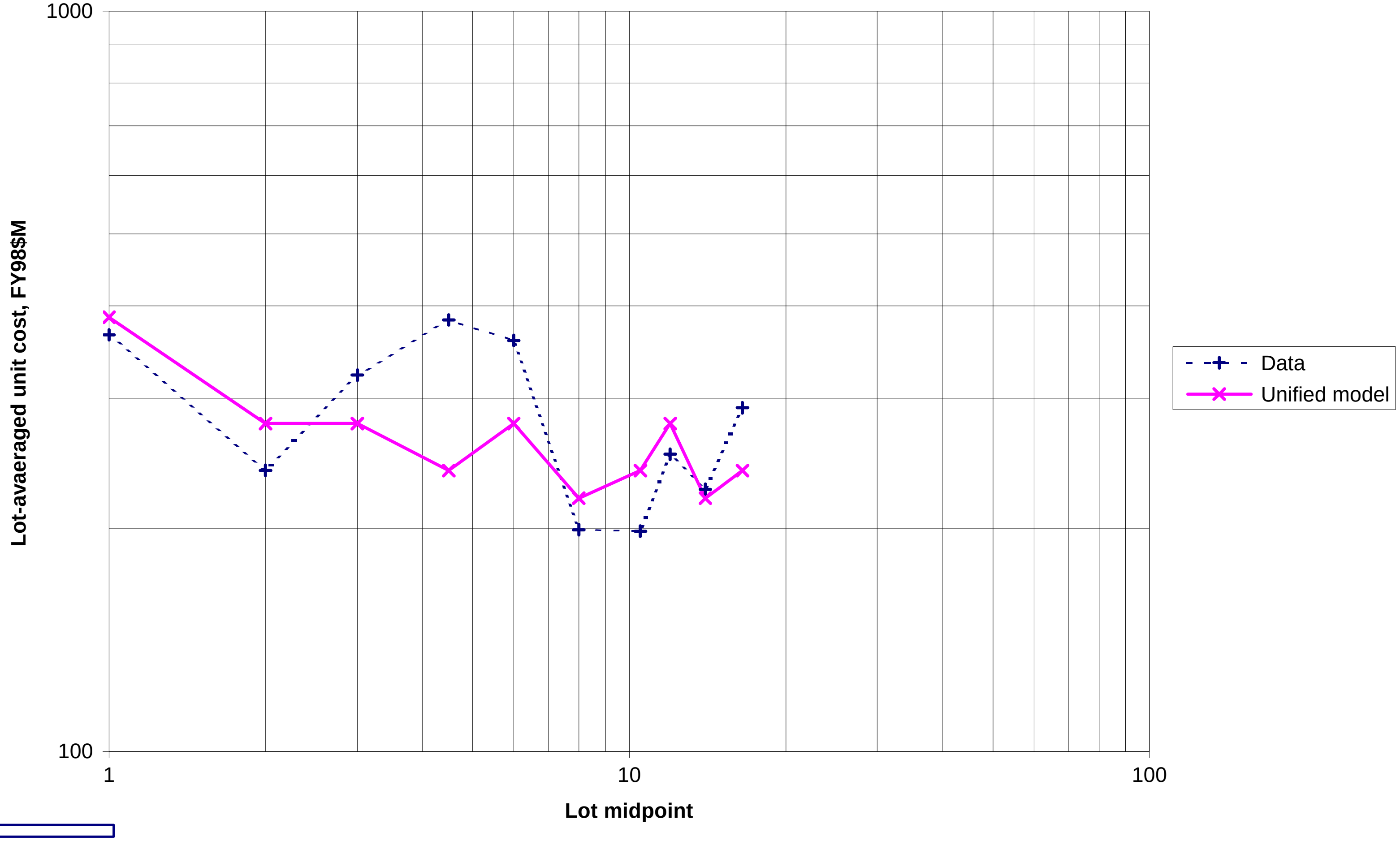
Results



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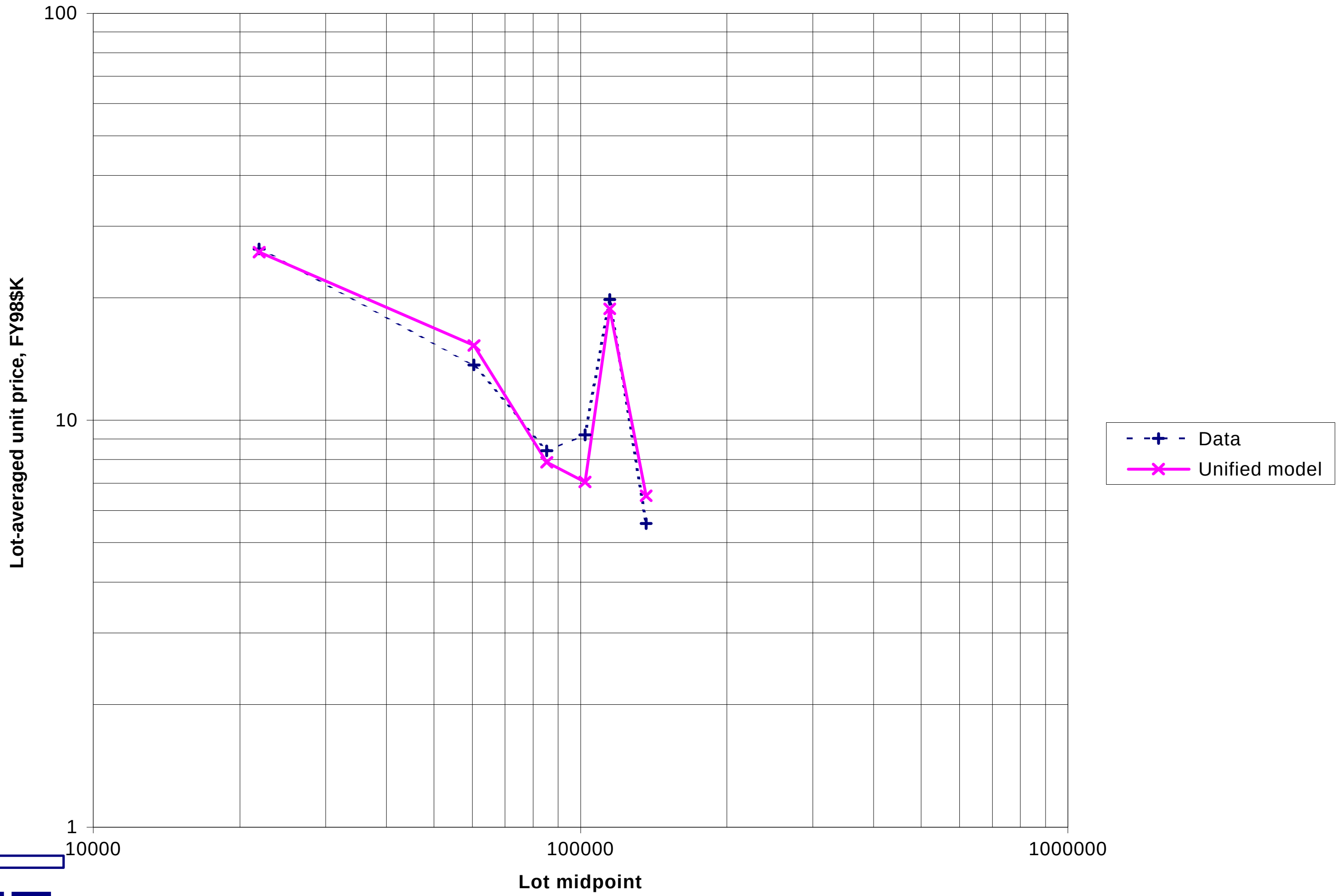
Results



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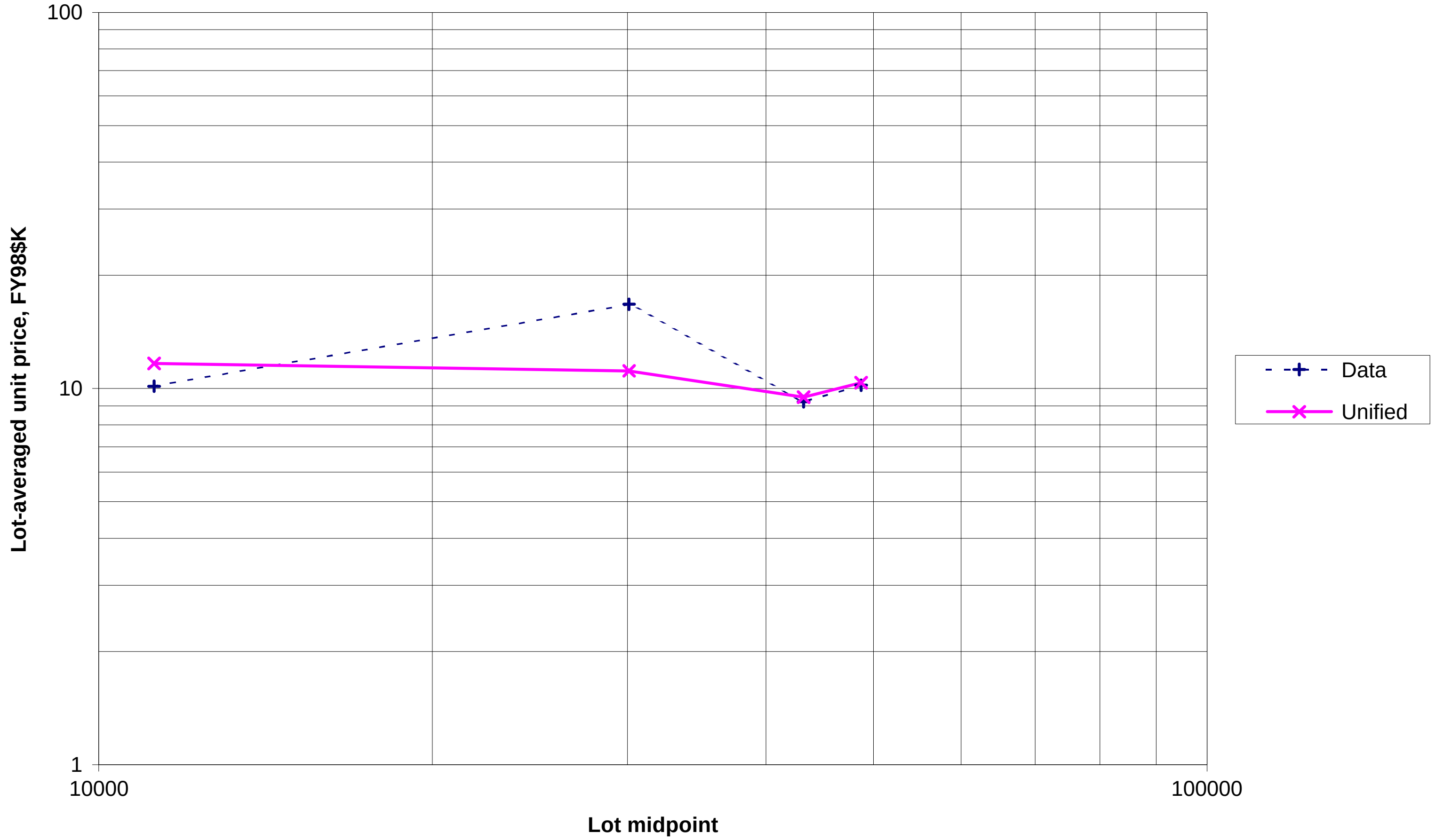
Results



100
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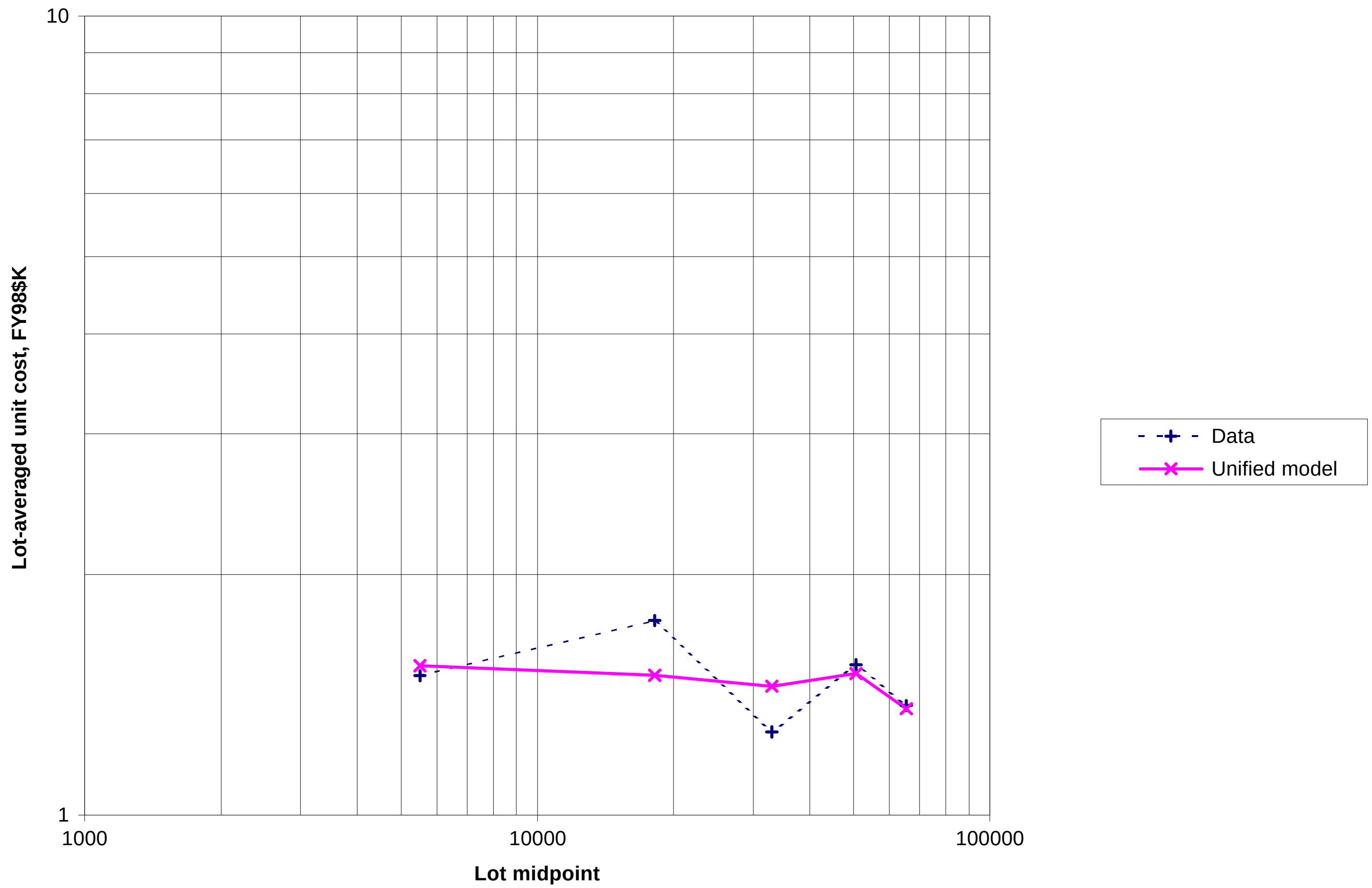
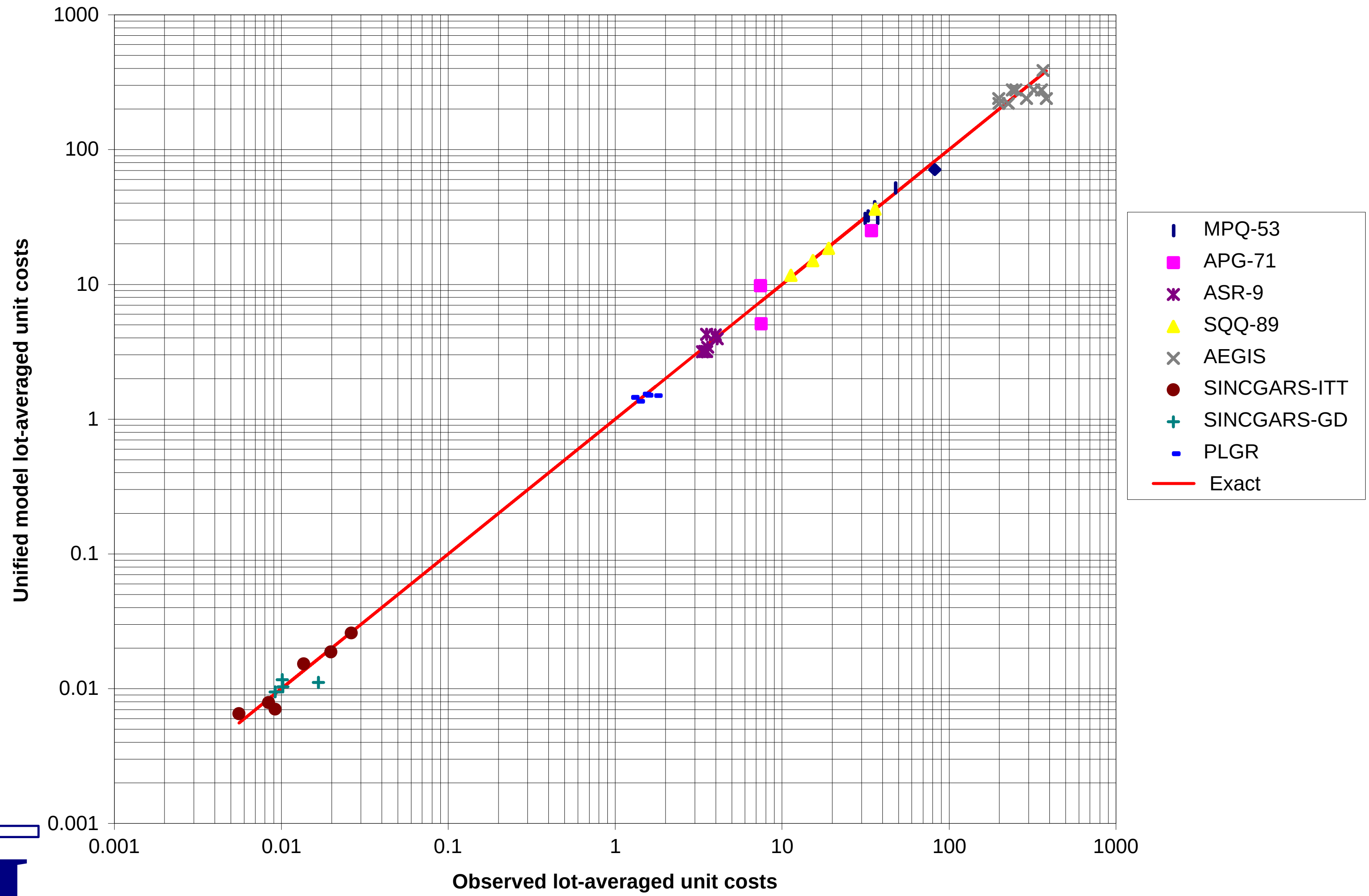
1
10000
100000
Lot-averaged unit price, FY98$K
Lot midpoint
Data
Unified

Results



10
1
1000
10000
100000
Lot-averaged unit cost, FY98$K
Lot midpoint
Data
Unified model

All Cases



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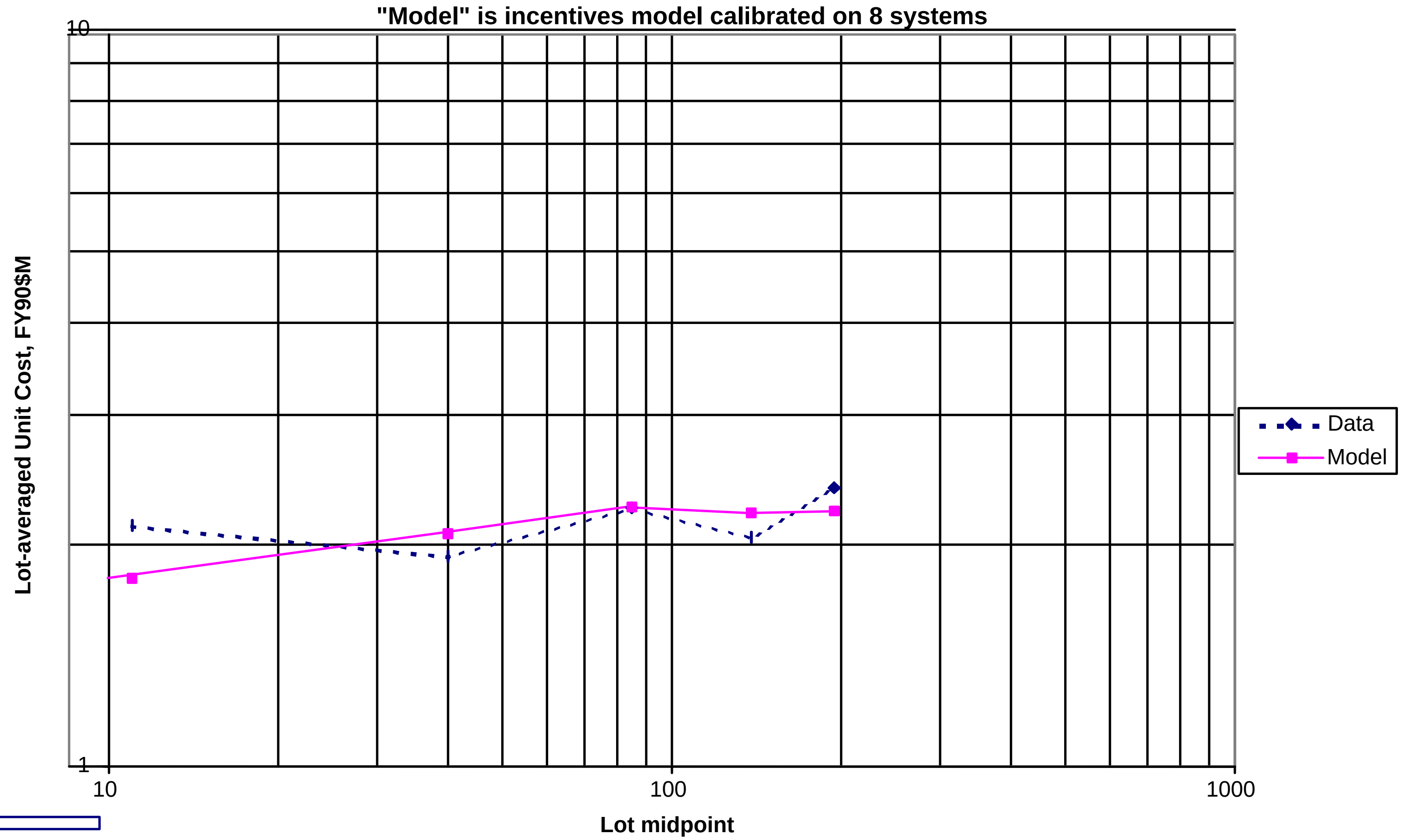
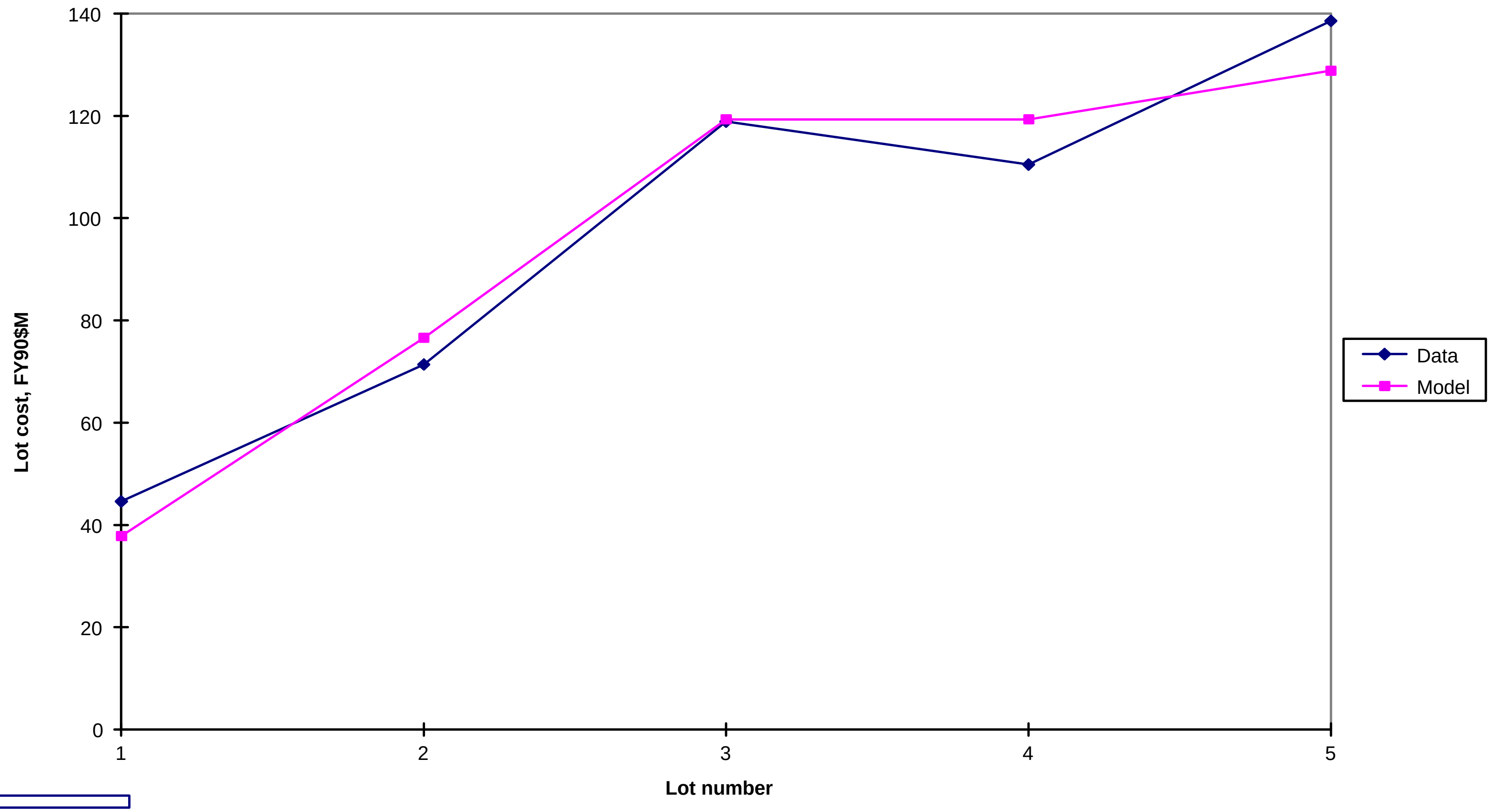
Application to a system not used in calibration



Application to system not used in calibration



Summary

- Straightforward descriptors of product and plant, used in an investment-incentives model of cost progress, explain variations in cost progress for widely differing electronics products and manufacturing environments relatively well
- Present model generates cost progress curve from answers to three questions, beside usual inputs of T_1 and quantity profile, and rate adjustment if that is desired
- Present model is just one member of a class. Its success encourages further exploration of the approach

How can I use this stuff?

- Qualitatively
  - Use characteristics of product and production environment (Complex product? Automated production? Competition? Product and environment conducive to productivity/production technology investment?) to select sets of old programs for use in deciding what cost progress to expect in a new program

How can I use this stuff?

- Quantitatively:
  - Fit appropriate 3-parameter model to data for appropriately selected set of old programs, use result numerically to forecast manufacturing cost stream for new program